

Clustering in the nuclear Fermi liquid

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We analyze the emergence of various structures in nucleonic matter, such as crystal, clusters, liquid drops and haloes. The formation of clusters indicates that nuclei behave like a Fermi liquid close to the liquid to solid transition. The relevant parameter is the ratio of the dispersion of the single-nucleon wave functions in the nucleus to the inter-nucleon distance. We also discuss the relationship between cluster states in nuclei and the pasta phase in the crust of neutron stars, as a transitional state between a Fermi liquid and a crystal. Haloes and clusters exhibit opposite features with respect to nucleonic localization.

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Nucleons in atomic nuclei and extended nuclear matter exhibit a variety of phases. Liquid drop aspects, for instance, were first inferred [1] from fission properties in heavy nuclei. Soon after it was also predicted and observed that cluster states could occur, especially in light nuclei [2]. Halo structures in nuclei were discovered in the late 1980's [3]. Although various theoretical models have been developed that successfully describe the corresponding characteristics of these states, open questions remain: can nucleonic crystal states occur?; do all nucleonic phases (liquid, cluster, halo, crystal) have a common origin and, therefore, can they be described in a unified theoretical framework? In a recent study [4] we have shown that the confining nuclear potential determines the degree of clustering in nuclei. In the present work we aim to further provide a unified view of the various nuclear states: cluster states should be considered as a transitional phase between a quantum liquid (nuclear matter) and a solid (crystal) and, in general, nuclei can be considered as liquid drops close to crystallization. These considerations are also relevant for the description of the crust of neutron stars, where it is known that decreasing matter density (further from the center of the star) leads to a transition from the nuclear matter phase (liquid) to a Wigner crystal, with a pasta (cluster) phase in between [5–7].

Clustering – the arrangement of nucleons in clusters of bosonic characters, especially in light nuclei, coexists with the nuclear mean-field. The mechanism that drives the transition from the fermionic liquid to the crystal-like cluster structures is not fully understood [8] but, as we have shown in the recent study of Ref. [4], its origin partly lies in the nuclear potential that confines the neutrons and protons to the nucleus. The deformation of a nucleus also plays an important role in the formation of clusters because it removes the degeneracy of single-nucleon levels associated with spherical symmetry [2]. Clustering predominantly appears in excited states, and this can be understood from the fact that closeness to the particle emission threshold favors cluster formation. States close to the continuum cannot be isolated from the environ-

ment of scattering states, so cluster states at the threshold belong to an open quantum system [9]. The origin of cluster formation, however, lies in the effective nuclear interaction, and a fully microscopic description of clustering necessitates a framework that encompasses both cluster and quantum liquid-drop aspects in light and heavier nuclei. In Ref. [4] we performed self-consistent mean-field calculations based on nuclear energy density functionals (EDFs) to explore the effect of the confining nuclear potential on the formation of localized structures in the equilibrium state of ^{20}Ne , and showed that the depth of the potential drives the energy spacings between single-nucleon orbitals in deformed nuclei, the localization of the corresponding wave functions and, therefore, the degree of nucleonic density clustering. The localization of the single-nucleon wave function can be quantified by the dimensionless parameter [4]:

$$\alpha \doteq \frac{\Delta r}{\bar{r}} \quad (1)$$

where \bar{r} is the average inter-nucleon distance and Δr the spatial dispersion of the wave function. When $\alpha \gtrsim 1$ nucleons are delocalized and the nucleus displays a quantum liquid structure. On the other hand, when $\alpha \lesssim 1$ nucleons are localized on the nodes of a crystal-like structure. The transition from the quantum liquid phase to cluster states occurs for $\alpha \simeq 1$. The s , p and d wave functions give the principal contribution to cluster states in light nuclei. The spatial dispersion of can be calculated from:

$$\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} \quad (2)$$

In Ref. [4] we used two functionals that are representative of the two standard classes of nuclear EDFs: the non-relativistic Skyrme SLy4 [10], and the relativistic functional DD-ME2 [11], to calculate the self-consistent equilibrium mean-field solution for ^{20}Ne . Even though both functionals reproduce the empirical ground-state properties (binding energy, charge radius, matter radius) with a typical accuracy of 1%, as well as the quadrupole deformation of the equilibrium shape, the density calculated

| | Self-cons. | | HO | |
|-------------------|------------|-------|-------|-------|
| | SLy4 | DDME2 | SLy4 | DDME2 |
| ²⁰ Ne | 0.987 | 0.971 | 0.996 | 0.973 |
| ²⁴ Mg | 1.001 | 0.948 | 1.015 | 0.963 |
| ²⁸ Si | 0.986 | 0.955 | 1.053 | 0.996 |
| ³² S | 0.991 | 0.961 | 1.057 | 0.993 |
| ²⁰⁸ Pb | 1.278 | 1.309 | 1.458 | 1.395 |

TABLE I. Left: the localization parameter α calculated from self-consistent equilibrium solutions determined using the SLy4 [10] and DDME2 [11] density functionals (Eqs. (1) and (2), see text). Right: the same but using the 3D harmonic oscillator approximation (Eq. (6)).

with SLy4 displays a smooth behavior characteristic of a Fermi liquid, whereas the functional DD-ME2 predicts an equilibrium density that is much more localized, with pronounced cluster structures. The difference in the calculated equilibrium density distributions was traced back to the depth of the confining potential: in the centre of the nucleus the depth of the DD-ME2 single-neutron potential is -79.7 MeV, whereas the depth of the SLy4 potential is -71.4 MeV. The corresponding values of the single-proton potentials are: -74.8 MeV for DD-ME2, and -66.5 MeV for SLy4.

Using the same functionals SLy4 and DD-ME2 to calculate the self-consistent equilibrium nuclear mean-field state, in the present work we have computed the spatial dispersion of the Nilsson state $[1\ 1\ 0\ 1/2]$ in the $N = Z$ nuclei ²⁰Ne, ²⁴Mg, ²⁸Si, ³²S, using Eq. (2). The dispersion of the same state is also calculated for ²⁰⁸Pb. Taking $\bar{r} = 0.9$ fm as a characteristic inter-nucleon equilibrium distance [12], we determine the corresponding values of the localization parameter α displayed in the first two columns of Table I. In light nuclei the values of α calculated with DD-ME2 are smaller than those obtained us-

ing SLy4, reflecting the more pronounced localization of the nucleonic densities that was already observed in our previous study in Ref. [4]. While for light nuclei $\alpha \leq 1$, in the case of ²⁰⁸Pb α is considerably larger than 1 and this unambiguously characterizes the quantum liquid phase of nucleonic matter in this nucleus.

In a non self-consistent approximation one can use a 3-dimensional isotropic harmonic oscillator (HO) for the the confining nuclear potential. This approximation allows for a further qualitative discussion of the effects of the effective nuclear interaction on the spatial arrangement of nucleons. The 3-dimensional HO wave functions $\varphi_{klm}(\vec{r})$ for the first s , p and d states read [13]:

$$\varphi_{0lm}(\vec{r}) \sim \frac{r^l}{b^{(3/2+l)}} e^{-\frac{r^2}{2b^2}} Y_l^m(\hat{r}), \quad (3)$$

where b is the oscillator length defined by:

$$b \doteq \sqrt{\frac{\hbar}{m\omega_0}} = \frac{\sqrt{\hbar R}}{(2mV_0)^{1/4}}. \quad (4)$$

R is the radius of the potential for which $V = 0$, and V_0 denotes the depth of the potential for $r = 0$. We note that charge radii of atomic nuclei are determined with high precision in electron scattering experiments, in contrast to the depth of the confining potential V_0 which is experimentally poorly constrained.

A straightforward calculation using Eqs. (2) and (3) yields $\Delta r \approx 0.5b$ for the first s , p and d states, and they display a Gaussian-like radial dependence (Eq. (3)). Consequently, for a constant radius R a deeper potential V_0 implies a smaller value of the oscillator length b (Eq. (4)), and thus a smaller dispersion. This effect is illustrated in Figure 3 of Ref. [4], which shows the radial part of the p wave function for two different values of the oscillator length b . A smaller value of b implies a more localized wave function, eventually leading to cluster formation.

This concept can be extended to the more general case of deformed nuclei by approximating the confining potential with an axially deformed HO. The wave functions are then expressed as [14, 15]

$$\varphi_{n_r, n_z, m_l}(r, \phi, z) \sim e^{im_l \phi} \left(\frac{r}{b_\perp} \right)^{m_l} H_{n_z}(z/b_z) L_{n_r}^{m_l}(r^2/b_\perp^2) e^{-\frac{1}{2} \left(\frac{z^2}{b_z^2} + \frac{r^2}{b_\perp^2} \right)}, \quad (5)$$

where H and L are the Hermite and Laguerre polynomials, respectively. Eq. (5) displays a radial dependence similar to the 3D isotropic case (Eq. (3)): the dispersion of the wave functions depends on the oscillator lengths b_z and b_\perp in the respective directions, which in turn depend on the depth of the potential. The effect of the depth of the potential on the localization of the nucleon wave functions is universal.

In the second two columns of Table I we list the val-

ues of α obtained using expression (4) for the harmonic oscillator length:

$$\alpha = \frac{b}{r_0} = \frac{\sqrt{\hbar R}}{r_0(2mV_0)^{1/4}}, \quad (6)$$

with R and V_0 determined microscopically using the SLy4 and DD-ME2 self-consistent equilibrium solutions, respectively, and $r_0 = 1.25$ fm [16]. The same trend is found as in the self-consistent calculation: α is smaller

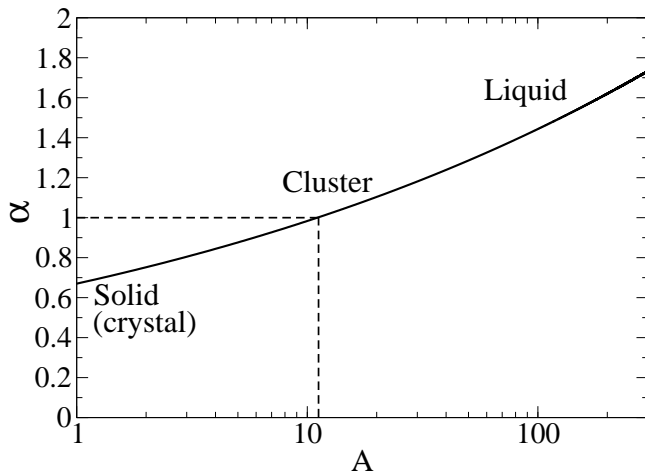


FIG. 1. The localization parameter α as a function of the number of nucleons (Eq. 7). The transition to cluster states occurs for $\alpha \simeq 1$.

in the case of DDME2, and $\alpha > 1$ for ^{208}Pb , indicating a quantum liquid behavior for this heavy nucleus.

The localization parameter α generally increases with the number of nucleons A : cluster states are more likely formed in light nuclei, as observed experimentally [2]. Using the liquid drop parameterization for the radius $R=r_0A^{1/3}$, Eq. (6) reads

$$\alpha = \frac{\sqrt{\hbar}A^{1/6}}{(2mV_0r_0^2)^{1/4}} \simeq 0.67A^{1/6} \quad (7)$$

Figure 1 displays the evolution of the parameter α with A , for typical values of $V_0 = 70$ MeV. Obviously clustering is favored in light nuclei, whereas the quantum liquid phase is predominant in heavy nuclei. Quantitatively, the transition from a cluster to a liquid state in nuclei is obtained around $A \approx 30$ (α between 0.8 and 1.2), and this is in agreement with the empirical observation that cluster states are more likely to occur in light nuclei. For heavier systems α is considerably larger than 1 and, therefore, cluster states are less likely to appear in heavy nuclei. It should also be noted that Fig.1 nicely illustrates why a crystal phase (α at least below 0.8) cannot occur in nuclei. However, as described below, nature may offer the possibility of existence of nucleonic crystals in neutron stars, due to gravitational and density conditions at work at these astrophysical sites.

The issue of solid (crystal) vs. quantum liquid nature of nuclei was already addressed by B. Mottelson, who introduced the *quantality* parameter [12]:

$$\Lambda \hat{=} \frac{\hbar^2}{m\bar{r}^2V_0'} \quad (8)$$

where V_0' is the strength of the bare nucleon-nucleon interaction (~ 100 MeV), and \bar{r} is inter-nucleon equilibrium distance. The quantality Λ is defined as the ratio

of the zero-point kinetic energy to the potential energy, and typical values obtained for nuclei are characteristic for a quantum liquid phase [12]. However, cluster states in nuclei introduce an additional phase of nucleonic matter. The parameter α introduced in this work appears more accurate in this respect than the *quantality* because α takes into account the nuclear medium effect on the potential, whereas Λ uses the one that corresponds the bare nucleon-nucleon interaction. The quantality may not contain information of the localization of the wave-function in the nuclear medium. Two characteristic lengths (Δr and \bar{r}) should be considered to quantify crystalline vs. liquid effects, as in condensed matter [17]: the localization of the constituent wave function in the system, and the inter-constituent distance.

An important ingredient to understand clustering is the repulsive part of the interaction which gives rise to a crystal lattice on which nucleons can be localized. For instance, in the crust of compact stars the Coulomb force plays the role of the repulsive interaction that frustrates the system and leads to nucleonic crystallization in the external low-density part of the crust. The so-called pasta phase, predicted in the inner part of the crust is considered as a transition between a liquid and a Wigner crystal phase [5, 6]. An analogy can be drawn between nuclear cluster states and the pasta phase for which the molecular dynamics approach is used [7]. Interestingly, in the seminal paper on pasta phase Baym et al. discussed about the necessity to investigate the transition between the solid and liquid phases in neutron star matter [5]. Lattimer and Swesty considered the ratio u of the volume of a nucleus to the volume of the Wigner-Seitz cell [6]. This quantity corresponds to the cube of the ratio of the nuclear size to the inter-nuclear distance, and this is analogous to the definition of α (Eq. (1)). In Ref. [6] $u = 1$ corresponds to nuclear matter and the pasta phase can already appear for $u = 0.72$. For lower values of u the crystal structure dominates. The transition between the Wigner crystal and the quantum liquid in the neutron star crust can be described by various models: gelification [18], Coulombic frustration [19] or quantum melting [20].

In the case of atomic nuclei the inter-nucleon interaction contains both an attractive and a repulsive component. Therefore the hard-core part of the nucleon-nucleon interaction generates a typical \bar{r} spatial scale. Most nuclei, of course, have $\alpha > 1$ in their Fermi liquid states, especially heavy ones as discussed above. Nuclei, therefore, consist of largely delocalized nucleons and this explain their liquid drop nature and also the large mean free path of nucleons. But, more precisely, nuclei are in the Fermi liquid state at the limit of crystallization, explaining why localized cluster states can be formed in light nuclei. The short range of the hard-core also explains why genuine nucleonic crystals cannot occur in nuclei, whereas they can appear in the crust of neutron stars where crystallization is caused by the long range Coulomb interaction.

Halo states are known to have delocalized wave functions [21] and could be related to the present discussion. When the confining nuclear potential is approximated by a square-well, the oscillations of the wave function of a state of energy E (<0) are determined by the wave number $\hbar^2 k^2 = 2m(E + V_0)$, whereas outside of the potential the decay of the wave function is governed by $\hbar^2 k'^2 = -2mE$. Inside the potential a larger k favors localization. This happens when the potential is deeper and/or E gets closer to 0: the degree of localization depends on the difference $E - (-V_0)$, and a deeper potential favors the localization of the wave function. Also a weakly-bound state will be more localized, and this means that cluster states are more likely to occur for weakly-bound states close to the emission threshold, in agreement with the Ikeda picture [22]. An important characteristic of nuclei is that quantitatively the dispersion of the single-nucleon wave function can be of the

same order of magnitude as the inter-nucleon distance, leading to clustering as discussed above. Outside of the potential the decay of a single-nucleon wave function is only determined by the energy of the state, and a weakly-bound state displays a large radial extension in the external region. In the limiting case of very weakly bound states, the pronounced delocalization of the corresponding wave functions outside of the potential gives rise to halo structures [21]. Inside the nuclear well, as discussed above, both the energy of a state and the depth of the potential play a role in the localization of the wave function. To summarize, the single-nucleon equation of motion relates energies (depth of the potential and/or binding energies) to the localization of the corresponding wave function. In nuclei the typical values of length scales and energies allow the formation of clusters, liquid drops and haloes.

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- [1] C.F. Von Weizsäcker, Zeitschrift für Physik 96 (1935) 431; H.A. Bethe and R.F. Bacher, Rev. Mod. Phys. 8 (1936) 82.
 - [2] W.v Oertzen, M. Freer, Y. Kanada-En'yo, Phys. Rep. 432, 43-113 (2006), and references therein.
 - [3] I. Tanihata et al., Phys. Rev. Lett. 55, 2676 (1985).
 - [4] J.P. Ebran, E. Khan, T. Niksic and D.Vretenar, Nature 487, 341 (2012).
 - [5] G. Baym, H.A. Bethe and C. Pethick, Nucl. Phys. A175, 225 (1971).
 - [6] J.M. Lattimer and F.D. Swesty, Nucl. Phys. A535, 331 (1991).
 - [7] G. Watanabe, T. Maruyama, K. Sato, K. Yasuoka and T. Ebisuzaki, Phys. Rev. Lett. 94, 031101 (2005).
 - [8] M. Freer, Nature 487, 309 (2012).
 - [9] J. Okolowicz, M. Ploszajczak, W. Nazarewicz, arXiv:1202.6290 (2012).
 - [10] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, R. Schaeffer, Nucl. Phys. A 635, 231-256 (1998).
 - [11] G.A. Lalazissis, T. Niksic, D. Vretenar, P. Ring, Phys. Rev. C 71, 024312 (2005).
 - [12] B. Mottelson, Nuclear Structure, Les Houches, Session LXVI, 25 (1996).
 - [13] C. Cohen-Tannoudji, B. Diu, F. Laloe, Mécanique Quantique, (Hermann Ed., 1973).
 - [14] S.G. Nilsson, Mat Fys Medd Dan.Vid. Selsk. 29, 16 (1955).
 - [15] G. Agala, Nucl. Phys. 4, 625 (1957).
 - [16] A. Bohr and B. Mottelson, *Nuclear structure* (Benjamin, New York, 1969).
 - [17] D. Pines, P. Noziers, The theory of quantum liquids, (Benjamin Inc., 1966).
 - [18] A. Coniglio et al., J. Phys.: Condensed Matter 18, S2383 (2006).
 - [19] P. Napolitani, Ph. Chomaz, F. Gulminelli and K.H.O Hasnaoui, Phys. Rev. Lett. 98, 131102 (2007).
 - [20] H. Falakshahi and X. Waintal, Phys. Rev. Lett. 94, 046801 (2005).
 - [21] P.G. Hansen and B. Jonson, Europhys. Lett. 4, 409 (1987).
 - [22] K. Ikeda, N. Tagikawa, H. Horiuchi, Prog. Theor. Phys. (Suppl.) 464 (1968).